Solution to MHT CET – 2021
22nd September (Shift - 2)

Section I

PHYSICS

1. (A)
Area under the F–t graph gives change in momentum. Since the body is initially at rest, it gives the momentum of the body after 1 second.

Area $= 10 \times 0.5 + 20 \times 0.5$
$= 5 + 10 = 15$ N–s

$\therefore\ mV = 15$ N–s

$$V = \frac{15}{m} = \frac{15}{3} = 7.5 \text{ m/s}$$

2. (D)
(I) NAND gate gives output '0' when inputs are 1, 1
(II) NOR gate gives output '1' when inputs are 0, 0
(III) AND gate gives output '0' when inputs are 0, 1
(IV) EX–OR gate gives output '1' when inputs are 1, 0
$\therefore$ Ans. II and IV

3. (B)
Power gain = (voltage gain) × (current gain)

$$\text{Current gain} = \frac{\text{output current}}{\text{Input current}} = \frac{\frac{V_0}{z_0}}{\frac{V_i}{z_i}}$$

$$= \frac{V_0}{V_i} \cdot \frac{z_i}{z_0} = \frac{40 \times 100}{400} = 10$$

Power gain = 40 × 10 = 400

4. (C)
For first resonance $L_1 = \frac{\lambda}{4}$

For second resonance $L_2 = \frac{3\lambda}{4}$

$\therefore\ L_2 - L_1 = \frac{\lambda}{2}$ or $\lambda = 2(L_2 - L_1)$

$V = n\lambda = 2n(L_2 - L_1)$

5. (D)
The field produced by charge on each plate is $\frac{E}{2}$.

Hence force on each plate is given by

$F = Q \times$(Field produced by the other plate)

$= \frac{QE}{2}$

6. (B)

The angular momentum of the particle is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

Its magnitude is $L = rp\sin\theta$

L will be maximum when $\theta = 90°$.

7. (B)

The ball is moving with constant velocity.

Hence the net force acting on it is zero.

The weight of the ball = $W = \frac{4}{3}\pi r^3 \rho_b g$ (in downward direction)

The viscous force F_v is in downward direction

The buoyant force $F_B = \frac{4}{3}\pi r^3 \rho_\ell g$ (in upward direction)

$$\therefore\ F_v + W = F_B$$

$$\therefore\ F_v = F_B - W$$

$$= \frac{4}{3}\pi r^3 g(\rho_\ell - \rho_b) = \frac{4}{3}\pi r^3 g \times 3\rho_b \quad (\because \rho_\ell = 4\rho_b)$$

$$\therefore\ \frac{F_v}{W} = 3$$

8. (A)

The magnetic field produced by a circular coil of radius r at its centre is given by

$$B = \frac{\mu_0 I}{2r}$$

In this case $r = 0.8$ m

Frequency $f = 1$ r.p.s.

$$\therefore\ I = qf = 100\,e\,A$$

$$\therefore\ B = \frac{\mu_0 \times 100\,e}{2 \times 0.8} = \frac{\mu_0 \times 100 \times 1.6 \times 10^{-19}}{1.6}$$

$$= 10^{-17}\mu_0$$

9. (B)

Kinetic energy of a satellite is given by

$$K.E. = \frac{GMm}{2r}$$

The two satellites are of the same mass

$$\therefore\ K.E. \propto \frac{1}{r}$$

$$\frac{(K.E.)_1}{(K.E.)_2} = \frac{r_2}{r_1} = \frac{3R}{2R} = \frac{3}{2}$$

10. (D)

Emissive power $E = \frac{Q}{At}$

$$\therefore\ Q = EAt = 0.7 \times 0.04 \times 20 = 0.56\ \text{kcal s}^{-1}\,\text{m}^{-2}$$

11. (D)
The internal resistance is given by

$$r = R\left(\frac{\ell_1-\ell_2}{\ell_2}\right)$$

$$= 2\left(\frac{240-120}{120}\right) = 2\times\frac{120}{120} = 2\,\Omega$$

12. (A)
f = 160 Hz, v = 320 m/s

$$\lambda = \frac{v}{f} = \frac{320}{160} = 2\text{ m} = 200\text{ cm}$$

Phase difference $\phi = \frac{2\pi x}{\lambda}$

$$\therefore\ x = \frac{\phi\lambda}{2\pi}$$

$$= \frac{\pi}{2}\cdot\frac{\lambda}{2\pi} \qquad \left[\because \phi = \frac{\pi}{2}\right]$$

$$= \frac{\lambda}{4} = \frac{200}{4} = 50\text{ cm}$$

13. (B)
Kinetic energy = – Total energy

14. (A)
Spheres are at the same potential

$$\therefore\ V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2}$$

$$\therefore\ \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

$$\sigma_1 = \frac{q_1}{4\pi r_1^2},\quad \sigma_2 = \frac{q_2}{4\pi r_2^2} \qquad \therefore\ \frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2}\cdot\frac{r_2^2}{r_1^2} = \frac{r_1}{r_2}\cdot\frac{r_2^2}{r_1^2}$$

$$\therefore\ = \frac{r_2}{r_1} = \frac{5}{4}$$

15. (C)

16. (A)
f = 500 Hz, v = 320 m/s

$$\lambda = \frac{v}{f} = \frac{320}{500} = 0.64\text{ cm}$$

Resonances will be obtained at air columns of lengths

$$\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \ldots\ldots$$

$$\frac{\lambda}{4} = \frac{64}{4} = 16\text{ cm}$$

∴ Resonance can be obtained at 16 cm, 48 cm, 80 cm, 105 cm, ….
Since the length of the tube is 100 cm, only first three resonances can be obtained.

17. (B)

Thermal resistance = $\frac{\text{Temp. difference}}{\text{Thermal current}} = \frac{28}{1400} = 0.02\ ^\circ\text{Cs/cal}$

18. (C)

The charge oscillates according to the equation

$$q = q_0 \cos \omega t$$

$$i = -\frac{dq}{dt} = \omega q_0 \sin \omega t = i_0 \sin \omega t$$

where $i_0 = \omega q_0,\ \omega = \frac{1}{\sqrt{LC}},\ q_0 = CV$

$$\therefore \quad i_0 = \frac{1}{\sqrt{LC}} \cdot CV = V\sqrt{\frac{C}{L}}$$

19. (A)

Energy of the photon is given by

$$E = h\nu$$

The frequency of photon does not change hence the energy does not change.

20. (A)

The intensity is given by

$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

Maximum intensity $K = 4I_0$ when $\phi = 0$

when path difference is $\frac{\lambda}{3}$, $\phi = \frac{2\pi}{3}$

$$\therefore \quad I = k\cos^2 \frac{2\pi}{3} = k\left(-\frac{1}{2}\right)^2 = \frac{k}{4}$$

21. (C)

Force on the electron $F = qE$

Work done by the force $W = qEL$

Work done is equal to gain in kinetic energy

$$\therefore \quad \frac{1}{2}mv^2 = qEL$$

$$\therefore \quad v = \sqrt{\frac{2qEL}{m}}$$

22. (B)

Terminal speed is given by

$$v = \frac{2}{9}\frac{r^2 g(\rho - \rho_L)}{\eta}$$

$$\therefore \quad \frac{V_A}{V_B} = \frac{\rho_A - \rho_L}{\rho_B - \rho_L} = \frac{7.5 - 1.5}{3 - 1.5} = \frac{6}{1.5} = 4$$

$$\therefore \quad V_B = \frac{V_A}{4} = \frac{0.4}{4} = 0.1\ \text{ms}^{-1}$$

23. (A)

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \qquad T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

When the two springs are connected in series, the effective spring constant is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

When connected in series

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\therefore \; T^2 = 4\pi^2\frac{m}{k} = 4\pi^2 m\left[\frac{1}{k_1} + \frac{1}{k_2}\right] = 4\pi^2\frac{m}{k_1} + 4\pi^2\frac{m}{k_2}$$

$$= T_1^2 + T_2^2$$

$$\therefore \; T = \sqrt{T_1^2 + T_2^2}$$

24. (A)

$$\Delta U = nC_v\Delta T$$

$$= n\frac{R}{\gamma - 1}\Delta T$$

$$= \frac{P\Delta V}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

$$\because \Delta V = 2V - V$$

25.(D)

Size of the image is equal to the size of the object. Hence image distance will be equal to object distance. Also if object distance $u = 2f$, Image distance will be 2f.

$$\therefore \; u = v = d = 2f$$

$$\therefore \; u + v = 2d = 4f$$

$$\therefore \; f = \frac{d}{2}$$

26. (A)

$$k = \frac{1}{2}I\omega^2$$

$$I' = 2I$$

By law of conservation of angular momentum

$$I'\omega' = I\omega$$

$$\therefore \; 2I\omega' = I\omega$$

$$\therefore \; \omega' = \frac{\omega}{2}$$

$$k' = \frac{1}{2}I'\omega'^2 = \frac{1}{2}(2I)\left(\frac{\omega}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{2}I\omega^2\right) = \frac{k}{2}$$

27. (D)

For isothermal process, $P_1V_1 = P_2V_2$

$$P_2 = P_1 - \frac{P_1}{10} = \frac{9}{10}P_1$$

$\therefore\ P_1V_1 = \frac{9}{10}P_1V_2$

$\therefore\ V_2 = \frac{10}{9}V_1$

$= 1.11\ V_1$

$= V_1 + 0.11V_1$

28. (D)

Refractive index of 'x' w.r.t. 'y' is

$${}_y n_x = \frac{1}{\sin\theta}$$

Also ${}_y n_x = \frac{V_y}{V_x} = \frac{1}{\sin\theta}$

$\therefore\ V_y = \frac{V_x}{\sin\theta}$

29. (A)

The weight of the needle is balanced by the force due to surface tension

$W = 2TL$

$= 2 \times 70 \times 7$

$= 980$ dyne

$= 1$ g wt

30. (A)

$u = 15$ m/s, $a = -0.3$ m/s^2, $t = 60$ s

$s = ut + \frac{1}{2}at^2$

$= 15 \times 60 + \frac{1}{2}\times(-0.3)\times(60)^2$

$= 900 - 540$

$= 360$ m

Distance from traffic light = 400 – 360 = 40 m

31. (B)

R.M.S. velocity is given by

$V = \sqrt{\frac{3RT}{M_0}}$

$\therefore\ V^2 = \frac{3RT}{M_0}$

$\therefore\ \frac{V^2}{T} = \frac{3R}{M_0} = \text{constant}$

32. (B)

Input power = I_1V_1

Output power = I_2V_2

33. (A)

The current through the inductor lags behind the applied emf by $\frac{\pi}{2}$ and the current through the capacitor leads the current by $\frac{\pi}{2}$

$$\therefore \; i_L = \frac{e_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) = -\frac{e_0}{\omega L}\cos\omega t$$

and $i_c = e_0\omega C\sin\left(\omega t + \frac{\pi}{2}\right) = e_0\,\omega C\cos\omega t$

34. (D)

We have $\frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - \phi = \frac{hc - \phi\lambda}{\lambda}$

$$\therefore \quad V_{max}^2 = \frac{2(hc - \phi\lambda)}{m\lambda}$$

$$\therefore \quad V_{max} = \sqrt{\frac{2(hc - \phi\lambda)}{m\lambda}}$$

35. (A)

$$i = \frac{V}{X_C} = V\omega C$$
$$= 230 \times 300 \times 100 \times 10^{-12}\ \text{Å}$$
$$= 6.9 \times 10^{-6}\ \text{Å}$$

36. (C)

Acceleration due to gravity is given by

$$g' = \frac{GM}{r^2} = \frac{GM}{(R+h)^2} = \frac{GM}{(2R)^2}$$
$$= \frac{GM}{4R^2} = \frac{g}{4}$$

37. (A)

$$I_1 = \frac{1}{2}MR^2,$$
$$I_2 = \frac{1}{2}(nM)(nR)^2 = \frac{1}{2}n^3MR^2$$
$$\therefore \; \frac{I_1}{I_2} = \frac{1}{n^3} = \frac{1}{8}$$
$$\therefore \; n^3 = 8 \text{ or } n = 2$$

38. (B)

Frequency of vibration of the string is given by

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2\rho}} = \frac{1}{2\ell r}\sqrt{\frac{T}{\pi\rho}}$$
$$\therefore \; \frac{n'}{n} = \frac{r}{r'} = \frac{1}{2}$$
$$\therefore \; n' = \frac{n}{2}$$

39. (B)

$\phi = 5t^2 - 6t + 9,\ t = 0.2\ s$

$e = -\frac{d\phi}{dt} = -(10t - 6)$

At $t = 0.2$ s, $|e| = 4$ V

$i = \frac{|e|}{R} = \frac{4}{20} = 0.2$ A

40. (C)

$E \propto \frac{1}{n^2}$ and $L \propto n$

$\therefore\ E \propto \frac{1}{L^2}$ or $E \propto L^{-2}$

41. (B)

Magnetic field inside the toroid is given by

$B = \mu_0 nI$

$\therefore\ \frac{B_2}{B_1} = \frac{I_2}{I_1} = 3$

(If cross–sectional radius of the solenoid is changed there will be no effect on the magnetic field, assuming the number of turns remains same.)

$\therefore\ B_2 = 3B_1 = 3 \times 0.2 = 0.6$ T

42. (C)

$x = P\sin\omega t + Q\sin\left(\omega t + \frac{\pi}{2}\right)$

It can be considered as composition of two S.H.M. of amplitudes P and Q having phase difference $\frac{\pi}{2}$.

$\therefore$ Resultant amplitude $R = \sqrt{P^2 + Q^2}$

Total energy $E = \frac{1}{2}m\omega^2 R^2$

$= \frac{1}{2}m\omega^2\left(P^2 + Q^2\right)$

43. (A)

If V is the potential difference then

$\frac{1}{2}mv_A^2 = qV$

and $\frac{1}{2}mv_B^2 = 4qV$

$\therefore\ \frac{v_A^2}{v_B^2} = \frac{1}{4}$

$\therefore\ \frac{v_A}{v_B} = \frac{1}{2}$

44. (C)

$\mu = \mu_r \mu_0$

$= 2000 \times 4\pi \times 10^{-7}$

$= 8\pi \times 10^{-4}$ SI unit

45. (B)

46. (B)

$V_s = 8V,\ R_s = 0.4\ \Omega$

$$I_s = \frac{V_s}{R_s} = \frac{8}{0.4} = 20\text{ A}$$

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{20}$$

$$\therefore I_p = \frac{I_s}{20} = \frac{20}{20} = 1\text{A}$$

47. (D)

Energy of photons, $h\nu = 10$ eV

Threshold frequency $= 2 \times 10^{15}$ Hz

$\therefore$ Work function, $W_0 = h\nu_0$

$= 6.63 \times 10^{-34} \times 2 \times 10^{15}$

$= 13.26 \times 10^{-19}$ J

$$= \frac{13.26\times10^{-19}}{1.6\times10^{-19}}\text{ eV}$$

$= 8.3$ eV

KE of electrons $= h\nu - W_0$

$= 10 - 8.3 = 2.7$ eV

48. (D)

Let N_1 be the number of nuclei of X_1 and

N_2 be the number of nuclei of X_2 after time t.

$\therefore N_1 = N_0 e^{-5\lambda t}$ and $N_2 = N_0 e^{-\lambda t}$

$$\therefore \frac{N_1}{N_2} = \frac{e^{-5\lambda t}}{e^{-\lambda t}} = \frac{1}{e^{4\lambda t}} = \frac{1}{e}$$

$\therefore 4\lambda t = 1$

$$\therefore t = \frac{1}{4\lambda}$$

49. (D)

If amplitudes are different, then in the region of destructive interference, the two waves do not cancel each other completely, hence there is some intensity or light.

50. (D)

For an adiabatic process $TV^{\gamma-1}$ = constant

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{27}{8}\right)^{\frac{5}{3}-1} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \frac{9}{4}$$

$$\therefore T_2 = \frac{4}{9}\cdot T_1 = \frac{4}{9}\times 300 = 675\text{ K}$$

$\therefore T_2 - T_1 = 675 - 300 = 375$ K

CHEMISTRY

51. (B)

$$\underset{\text{Ether}}{R-O-R'} + H_2SO_4 \longrightarrow \underset{\text{Oxonium salt}}{R-\overset{\overset{H}{|}}{\underset{\oplus}{O}}-R'\ HSO_4^{\ominus}}$$

52. (D)

$T_1 = 0°C = 273\ K, \quad V_1 = 2\ dm^3$

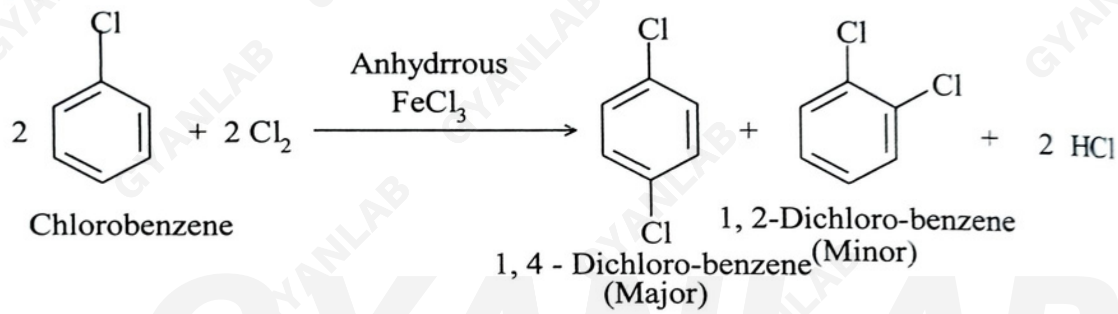
$T_2 = -272°C = 1K, \quad V_2 = ?$

According to Charle's law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \therefore\ V_2 = \frac{V_1 \times T_2}{T_1}$$

$$\therefore\ \frac{2\ dm^3 \times 1\ K}{273\ K} = \left(\frac{2}{273}\right) dm^3$$

53. (B)



54. (B)

$$\text{No. of moles of urea} = \frac{\text{Mass of urea}}{\text{Molar mass of urea}} = \frac{5.4}{60} = 0.09 \text{ moles}$$

55. (B)

56. (B)

$$\underset{\text{Propan}-1-\text{ol}}{CH_3-CH_2-CH_2-OH} \xrightarrow[623\ K]{Al_2O_3} \underset{\underset{(A)}{\text{Propene}}}{Ch_3-CH=CH_2} \xrightarrow[\text{(ii)}\ H_2O]{\text{(i) Conc.}\ H_2SO_4} \underset{\underset{(B)}{\text{Propan}-2-\text{ol}}}{CH_3-\underset{\underset{OH}{|}}{C}H-CH_3}$$

57. (A)

58. (C)

IF_3 – Yellow powder

59. (D)

$K_{sp} = 4.9 \times 10^{-13}, \ S = ?$

$AgBr \rightleftharpoons Ag^+ + Br^-$

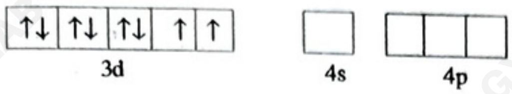
$\therefore\ K_{sp} = S^2 \qquad \therefore\ S = \sqrt{K_{sp}}$

$\therefore\ S = \sqrt{49 \times 10^{-14}} = 7.0 \times 10^{-7}\ mol\ dm^{-3}$

60. (C)
All lanthanoids are non-radioactive except promethium (Pm).

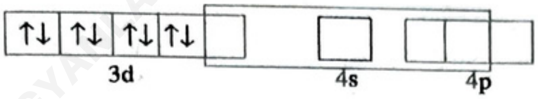
61. (B)
Structure of $[Ni(CN)_4]^{2-}$:
It is an example of square planar complex. The hybridization involved is dsp^2. Nickel is in +2 oxidation state. It has electronic configuration $3d^8$. The hybridization scheme is shown in diagram.

Orbitals of Ni^{2+} ion

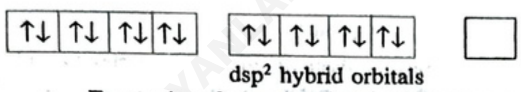


Since CN^- is a strong ligand, one of the unpaired electrons in 3d orbital is promoted giving two paired electrons and one vacant 3d orbital.

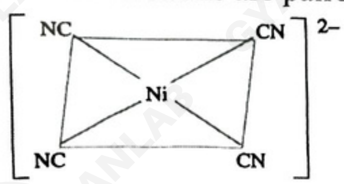
One 3d, one 4s and two 4p orbitals undergoing dsp^2 hybridization of Ni^{2+}



$[Ni(CN)_4]^{2-}$



Each of the hybridized orbitals receives a pair of electrons from a cyanide ion. The resulting complex ion is diamagnetic, since all the electrons are paired.



Structure of $[Ni(CN)_4]^{2-}$

62. (D)

$$N_{2(g)} + 2O_{2(g)} \longrightarrow 2NO_{2(g)}$$

$\Delta_{n(g)} = 2 - 3 = -1$

$\Delta H = \Delta U + \Delta n_{(g)} RT$

$\therefore$ $\Delta H > 0$ (for endothermic process)

63. (B)

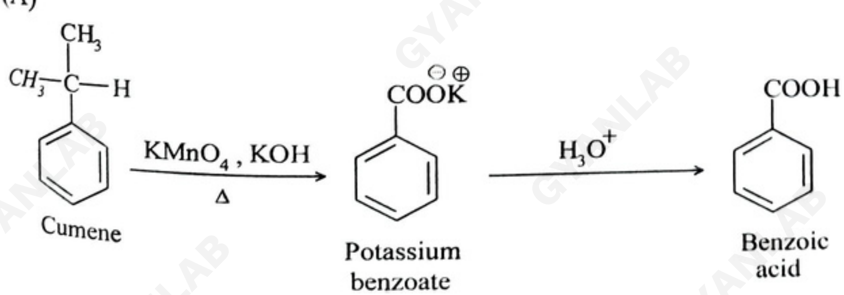
$$H_2\overset{-1}{O}_{2(g)} + \overset{+7}{Cl}O^-_{4(aq)} \longrightarrow \overset{+3}{Cl}O^-_{2(aq)} + \overset{0}{O}_{2(g)}$$

Oxidation number of O atom increases from –1 to 0. Hence it undergoes oxidation.

$\therefore$ H_2O_2 acts as reducing agent.

64. (A)

65. (A)



66. (D)

$$\propto = \sqrt{K_a . V}$$

67. (B)

$Ca^{2+} + 2e^- \rightarrow Ca$

40 g of Ca requires 2 F of electricity

$\therefore$ 0.8 g of Ca requires $= \dfrac{2F \times 0.8\,g}{40\,g} = 0.04$ F of electricity.

68. (B)

$$CH_3 - \overset{\delta_3}{C}H_2 \rightarrow \overset{\delta_2}{C}H_2 \rightarrow \overset{\delta_1}{C}H_2 \rightarrow \overset{\delta}{Cl}$$

Where, $\delta_3^{\oplus} < \delta_2^{\oplus} < \delta_1^{\oplus}$

69. (D)

For first order reaction,

$$k = \frac{2.303}{t} \log_{10} \frac{[A]_0}{[A]_t}$$

$$k = \frac{2.303}{23.03\text{ min}} \log_{10} \frac{0.08}{0.02}$$

$$= \frac{2.303}{23.03\text{ min}} \log_{10} 4 = 0.1\text{ min}^{-1} \times 0.6021$$

$$\therefore k = 0.06021\text{ min}^{-1}$$

70. (A)

Europium $(Eu_{63}) = [X_e]\, 4f^7\, 6s^2$

71. (D)

Volume of 2.43 g of element $= \dfrac{\text{Mass}}{\text{Density}} = \dfrac{2.43\,g}{9\,g\,cm^{-3}} = 0.27\,cm^3$

Number of unit cells $= \dfrac{\text{Total volume}}{\text{volume of a unit cell}} = \dfrac{0.27\,cm^3}{2.7 \times 10^{-23} cm^3} = 1 \times 10^{22}$

For BCC structure, 1 unit cell = 2 atoms

$\therefore$ 1×10^{22} unit cells $= 2 \times 10^{22}$ atoms

72. (A)

$$\underset{\text{Glucose}}{CHO-(CHOH)_4-CH_2OH} \xrightarrow{HI, \Delta} \underset{\text{n – Hexane}}{CH_3-(CH_2)_4-CH_3}$$

73. (C)

Buna-S is an elastomer which is a copolymer of styrene with butadiene. Its trade name is SBR (styrene-butadiene rubber).

$$\underset{\substack{\text{1, 3 - butadiene} \\ (75\ \%)}}{CH_2 = CH - CH = CH_2} + \underset{\substack{\text{styrene} \\ (25\ \%)}}{C_6H_5CH = CH_2}$$

$$\downarrow$$

$$\sim CH_2 - CH = CH - CH_2 - CH_2 - CH = CH - CH_2 - CH_2 - \underset{}{\overset{C_6H_5}{\overset{|}{CH}}} - CH_2 - CH_2 - CH = CH \sim$$

(SBR)

74. (A)
$W_2 = 5$ g, $M_2 = 342$ g mol^{-1}, $W_1 = 100$g, $\Delta T_f = 2.15$ K, $K_f = ?$

$$\Delta T_f = K_f \frac{1000\, W_2}{M_2 W_1}$$

$$\therefore\ K_f = \frac{\Delta T_f\, M_2\, W_1}{1000\, W_2}$$

$$\therefore\ K_f = \frac{2.15\,K \times 342\,g\,mol^{-1} \times 100\,g}{1000\,g\,kg^{-1} \times 5\,g}$$

$$\therefore\ K_f = 14.7\ K\ kg\ mol^{-1}$$

75. (B)
$p = 1$ bar, $S = 7 \times 10^{-4}$ mol L^{-1}

$$K_H = \frac{S}{P} = \frac{7 \times 10^{-4}\ mol\ L^{-1}}{1\ bar}$$
$$= 7 \times 10^{-4}\ mol\ L^{-1}\ bar^{-1}$$

76. (B)
$$\propto_1 = 10\% = \frac{10}{100} = 0.1,\ c_1 = 0.05\ M$$
$$c_2 = 0.10\ M,\ \propto_2 = ?$$
$$K_a = \propto_1^2 c_1 = \propto_2^2 c_2$$
$$\therefore\ (0.1)^2 \times 0.05 = \propto_2^2 \times 0.10$$
$$\therefore\ \propto_2^2 = \frac{10^{-2} \times 5 \times 10^{-2}}{10^{-1}} = 50 \times 10^{-4}$$
$$\therefore\ \propto_2 = \sqrt{50 \times 10^{-4}} = 7.17 \times 10^{-2} = 7.17\%$$

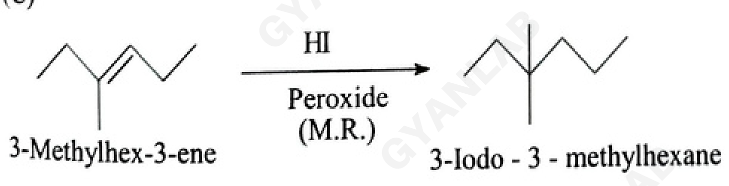
77. (B)
$C = 0.02$ M, cell constant $= 1.29$ cm^{-1}

$R_{solution} = 645\ \Omega$, $k = ?$

$$k = \frac{\text{cell constant}}{R_{solution}} = \frac{1.29\ cm^{-1}}{645\ \Omega}$$

$$\therefore\ k = 2.0 \times 10^{-3}\ \Omega^{-1}\ cm^{-1}$$

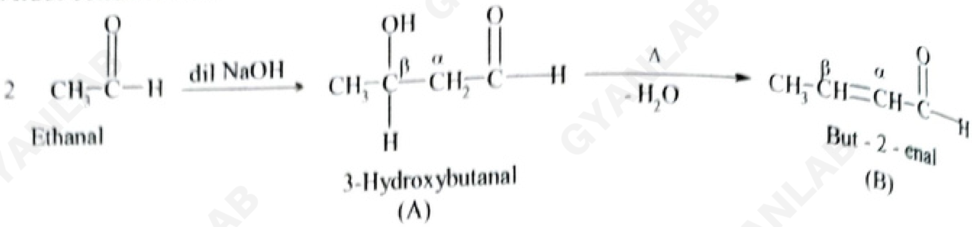
78. (C)



79. (B)
$r = k\,[H_2]\,[I_2]$
The reaction is first order in H_2 and I_2. The overall order of reaction is 2.

80. (C)

Aldol condensation



81. (C)

According to Hardy – Schulze rule, greater the valence of the flocculating ion added, the grater is its power to cause precipitation.

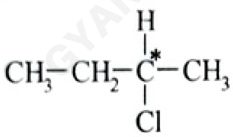
$\therefore Al^{3+} > Ba^{2+} > Na^{+}$

82. (C)

83. (C)

Buna – S is a copolymer while others are homopolymers.

84. (D)



Due to chiral carbon atom, it undergoes racemization during alkaline hydrolysis by S_N1 reaction.

85. (C)

$$N_{2(g)} + 3H_{2(g)} \longrightarrow 2NH_{3(g)}$$

$\Delta_r H^0 = \Sigma\Delta H^\circ$ (reactant) – $\Sigma\Delta H^\circ$(product)

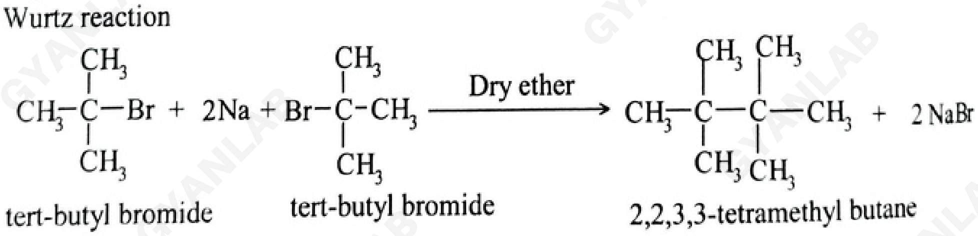
$= [\Delta H^\circ (N \equiv N) + 3\,\Delta H^\circ (H-H)] - [6\,\Delta H^\circ (N-H)] = 941 + 3\,(436) - 6\,(389)$

$\Delta_r H^\circ = -85$ kJ

$\therefore$ Enthalpy of formation of $NH_3 = -\dfrac{85}{2} = -42.5$ kJ

86. (A)

Wurtz reaction



87. (C)

88. (B)

$$N_{2(g)} + 3H_{2(g)} \longrightarrow 2\,NH_{3(g)}$$

Rate of reaction $= -\dfrac{d[N_2]}{dt} = -\dfrac{1}{3}\dfrac{d[H_2]}{dt} = \dfrac{1}{2}\dfrac{d[NH_3]}{dt}$

$\therefore \dfrac{d[H_2]}{dt} = 3\dfrac{d[N_2]}{dt}$

89. (C) **90.** (B)

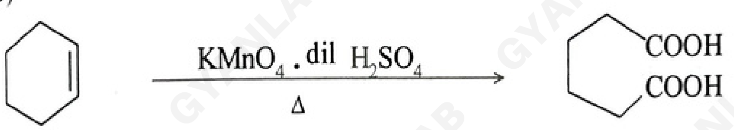
91. (A)
If N denotes the number of particles, then number of tetrahedral voids = 2 N
1 mole of a compound = 6.022×10^{23} particles
$\therefore$ 0.4 mole of a compound = $0.4 \times 6.022 \times 10^{23} = 2.4088 \times 10^{23}$ particles.
$\therefore$ No. of tetrahedral voids = $2 \times 2.4088 \times 10^{23} = 4.817 \times 10^{23}$

92. (D) **93.** (A)

94. (B)
p – Nitrophenol has highest melting point due to strong intermolecular hydrogen bonding.

95. (C)
$(CH_3)_2NH$ is a secondary amine and secondary amines are the strongest bases.

96. (D)



Cyclohexene Adipic acid

97. (B)

$$2C_2H_{6(\ell)} + 15O_{2(g)} \longrightarrow 12CO_{2(g)} + H_2O_{(\ell)}$$

$T = 298$ K, $R = 8.314\ J\ K^{-1}\ mol^{-1}$

$\Delta n_{(g)} = n_p - n_r = 12 - 15 = -3$ mol

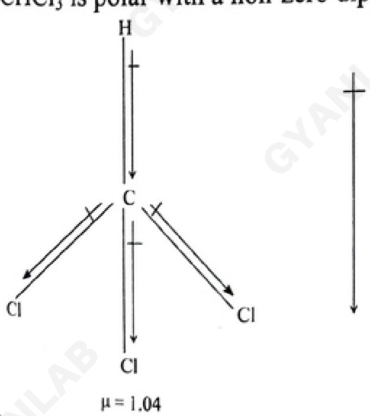
$\Delta H - \Delta U = \Delta n_{(g)}RT$

$= -3\ mol \times 8.314\ J\ K^{-1}\ mol^{-1} \times 298\ K$

$= -7432.716\ J = -7.43\ kJ$

98. (B)
Energy of the stationary state corresponding to n = 2 is
$E_2 = -2.18 \times 10^{-18}(1/(2)^2) = -0.545\ 10^{-18}$ J

99. (C)
$CHCl_3$ is polar with a non-zero dipole moment.



100. (A)
$[Co(NH_3)_6]^{3+}$ – Here only one type of ligands surrounds the Co^{3+} ion, hence it is homoleptic complex.

Section II

MATHEMATICS

101.(D)

$$\lim_{x\to\frac{\pi^-}{4}} f(x) = \lim_{x\to\frac{\pi^-}{4}} x + a\sqrt{2}\sin x$$

$$= \frac{\pi}{4} + a\sqrt{2}\sin\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + a \qquad \ldots(1)$$

$$\lim_{x\to\frac{\pi^+}{4}} f(x) = \lim_{x\to\frac{\pi^+}{4}} 2x\cot x + b$$

$$= 2\left(\frac{\pi}{4}\right)\cot\left(\frac{\pi}{4}\right) + b = \frac{\pi}{2} + b \qquad \ldots(2)$$

$$\lim_{x\to\frac{\pi^-}{2}} f(x) = \lim_{\frac{\pi^-}{2}} 2x\cot x + b$$

$$= 2\left(\frac{\pi}{2}\right)(0) + b = b \qquad \ldots(3)$$

$$\lim_{x\to\frac{\pi^+}{2}} f(x) = \lim_{x\to\frac{\pi^+}{2}} a\cos 2x - b\sin x$$

$$= a\cos 2\left(\frac{\pi}{2}\right) - b\sin\left(\frac{\pi}{2}\right)$$

$$= -a - b \qquad \ldots(4)$$

Since f(x) is continuous at $\frac{\pi}{4}$ and $\frac{\pi}{2}$, we write

$\frac{\pi}{4} + a = \frac{\pi}{2} + b$ …[From (1) and (2)]

$\therefore\ a - b = \frac{\pi}{4}$ …(5)

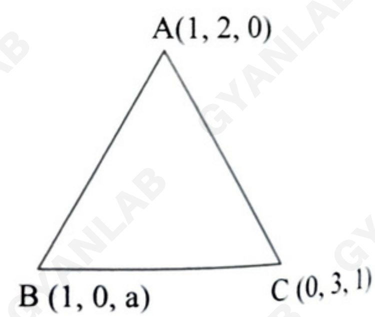
$b = -a - b$ …[From (3) and (4)]

$\therefore\ 2b = -a$ …(6)

Solving (5) and (6), we get $a = \frac{\pi}{6}, b = \frac{-\pi}{12}$

102.(B)

Refer figure



$A(\Delta ABC) = \frac{1}{2}\left|\overline{BA} \times \overline{BC}\right|$

Here $\overline{BA} = 2\hat{j} - a\hat{k}$

$\overline{BC} = -\hat{i} + 3\hat{j} + (1-a)\hat{k}$

Now $\overline{BA} \times \overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -a \\ -1 & 3 & 1-a \end{vmatrix}$

$= \hat{i}(2 - 2a + 3a) - \hat{j}(-a) + \hat{k}(2) = (a+2)\hat{i} + (a)\hat{j} + 2\hat{k}$

$\therefore\ |\overline{BA}\times\overline{BC}| = \sqrt{(a+2)^2+(a)^2+(2)^2}$

From given data, we write

$\sqrt{6} = \frac{1}{2}\sqrt{2a^2+4a+8}$

$\therefore\ 4(6) = 2a^2+4a+8 \Rightarrow a^2+2a-8=0 \Rightarrow (a+4)(a-2)=0 \Rightarrow a=-4, 2$

103.(D)

We have $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A(\text{adj } A) = AA^T$

$\therefore \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$\begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix} = \begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 9+4 \end{bmatrix}$

$\therefore\ 10a+3b = 13$ and $15a-2b=0$

Solving these equations, we get $a = \frac{2}{5}$ and $b = 3$

$\therefore\ 5a+b = 2+3 = 5$

104.(B)

We have 4 odd digits i.e. 1, 1, 3, 3 and 3 even digits i.e. 2, 2, 4

In a 7 digit number, there are 4 odd and 3 even places.

So number of possible ways $= \frac{4!}{2!2!}\times\frac{3!}{2!} = 18$

105.(C)

$y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$

$= \tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}} = \tan^{-1}\left(\cot\frac{x}{2}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right] = \frac{\pi}{2}-\frac{x}{2}$

$\therefore\ \frac{dy}{dx} = 0-\frac{1}{2} = \frac{-1}{2}$

106.(C)

We know that Coefficient of Variation $= \frac{\text{Standard Deviation}}{\text{Mean}}$

$\therefore\ (C.V.)_{Physics} = \frac{3}{20} = 0.15$ and $(C.V.)_{chemistry} = \frac{2}{25} = 0.08$

$(C.V.)_{Mathematics} = \frac{4}{23} = 0.174$ and $(C.V.)_{Biology} = \frac{5}{27} = 0.185$

107.(D)

We have vertices A(a, 3 1); B(4, 5, b); C(6, c, 5) and centroid G(4, 3, 3) of Δ ABC.

$$\therefore \quad \frac{a+4+6}{3}=4,\ \frac{3+5+c}{3}=3,\ \frac{1+b+5}{3}=3 \quad \Rightarrow \quad a=2, c=1, b=3$$

108.(A)

Equation of the plane passing through the line of intersection of the given planes, is
$(x + 2y + 3z - 4) + \lambda(4x + 3y + 2z - 1) = 0$

$\therefore \quad (1 + 4\lambda)x + (2 + 3\lambda)y + (3 + 2\lambda)z + (-4 - \lambda) = 0$ ...(1)

Since the plane (1) passes through origin,

We get $-4 - \lambda = 0 \quad \Rightarrow \quad \lambda = -4$

Substituting value of λ in equation (1), we get

$-15x - 10y - 5z = 0 \quad \Rightarrow \quad 3x + 2y + z = 0$

$\therefore$ d.r.s. are (3, 2, 1).

109.(A)

We have $y^2 = 8ax + 8a^2$ ...(1)

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 8a \quad \Rightarrow a = \left(\frac{y}{4}\right)\frac{dy}{dx}$$

Substituting value of 'a' in eq. (1), we get

$$y^2 = 8\left(\frac{y}{4}\right)\frac{dy}{dx}(x) + 8\left[\left(\frac{y}{4}\right)\left(\frac{dy}{dx}\right)\right]^2 = 2xy\frac{dy}{dx} + \left(\frac{y^2}{2}\right)\left(\frac{dy}{dx}\right)^2$$

$$\therefore \quad 2y^2 = 4xy\left(\frac{dy}{dx}\right) + y^2\left(\frac{dy}{dx}\right)^2$$

Hence order = 1, degree = 2

110.(A)

We have $ax^2 + 8xy + 5y^2 = 0$

Here $m_1 + m_2 = \frac{-8}{5},\ m_1m_2 = \frac{a}{5}$

As per condition given, we write

$$\left(-\frac{8}{5}\right) = 2\left(\frac{a}{5}\right) \quad \Rightarrow a = -4$$

111.(C)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \quad \text{...(let)}$$

$\therefore \quad a = k\sin A,\ b = k\sin B,\ c = k\sin C$

$$\therefore \quad \frac{b\sin B - c\sin C}{\sin(B-C)} = \frac{k\sin^2 B - k\sin^2 C}{\sin(B-C)} = \frac{k[\sin^2 B - \sin^2 C]}{\sin(B-C)}$$

$$= \frac{k\sin(B-C)\sin(B+C)}{\sin(B-C)}$$

$$= k\sin(B+C) = k[\sin(\pi - A)] = k\sin A = a$$

112.(C)

Equation of common chord is $S_1 - S_2 = 0$, where S_1 and S_2 are equations of the circles

$\therefore \left[(x-2)^2+(y-3)^2\right]-\left[(x-4)^2+(y-5)^2\right]=0$

$\therefore -4x + 4 - 6y + 9 + 8x - 16 + 10y - 25 = 0$

$\therefore 4x + 4y - 28 = 0 \Rightarrow x + y = 7$

113.(C)

$f(x)=\dfrac{\lambda \sin x + 6\cos x}{2\sin x + 3\cos x}$

$\therefore f'(x) = \dfrac{\{[(2\sin x+3\cos x)(\lambda\cos x-6\sin x)]-[(\lambda\sin x+6\cos x)(2\cos x-3\sin x)]\}}{(2\sin x+3\cos x)^2}$

When $f'(x) \ge 0$, we get

$[(2\lambda \sin x\cos x + 3\lambda\cos^2 x - 12\sin^2 x - 18\sin x\cos x) - (2\lambda\sin x\cos x + 12\cos^2 x - 3\lambda\sin^2 x - 18\sin x\cos x)] \ge 0$

$\therefore 3\lambda(\sin^2 x + \cos^2 x) - 12(\sin^2 x + \cos^2 x) \ge 0$

$\therefore 3\lambda - 12 \ge 0 \Rightarrow \lambda \ge 4$

114.(B)

$f(x)= x^2 + ax + b$

$\therefore f'(x) = 2x + a$ and when $f'(x) = 0$, we get $x = \dfrac{-a}{2}$

Now $f''(x) = 2$ and $2 > 0$

$\therefore f(x)$ has minima at $x = \dfrac{-a}{2} = 3$ ...[as per given data]

$\therefore a = -6$

Since minimum value of f(x) is 5 at x = 3, we write

$5 = (3)^2 + (-6)(3) + b \Rightarrow b = 14$

115.(A)

The point of intersection of $y^2 = x$ and $x + y = 2$ is,

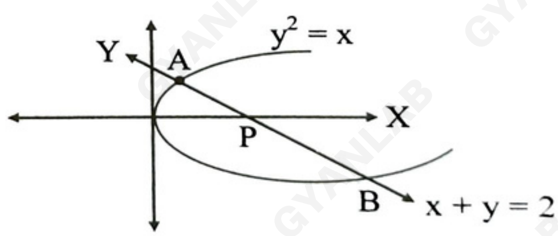
$(2-x)^2 = x \Rightarrow x^2 - 5x + 4 = 0 \Rightarrow (x-4)(x-1) = 0$

Let A = (1, 1) in first quadrant and B = (4, –2) in fourth quadrant

The line x + y = 2 cuts X axis at P(2, 0)

Refer figure

Required area is shaded.



$\therefore A = \int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)\,dx$

$= \left[\dfrac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^1 + [2x]_1^2 - \left[\dfrac{x^2}{2}\right]_1^2$

$= \left[\left(\dfrac{2}{3}\right)(1)\right] + [2(2-1)] - \left[\left(\dfrac{4-1}{2}\right)\right] = \dfrac{2}{3} + 2 - \dfrac{3}{2} = \dfrac{7}{6}$ sq. units

116.(D)

We have $\cos\theta = \frac{1}{2}\left(x+\frac{1}{x}\right)$

$2\cos 3\theta$

$= 2[4\cos^3\theta - 3\cos\theta] \quad = 2\left\{4\left[\left(\frac{1}{2}\right)\left(x+\frac{1}{x}\right)\right]^3 - \left[3\left(\frac{1}{2}\right)\left(x+\frac{1}{x}\right)\right]\right\}$

$= 2\left[\left(\frac{1}{2}\right)\left(x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)\right)\right] - 3\left(x+\frac{1}{x}\right) \quad = x^3+\frac{1}{x^3}$

117.(D)

We have $A(\text{adj } A) = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$

$\therefore \quad |A|\,|\text{adj } A| = \begin{vmatrix} 20 & 0 \\ 0 & 20 \end{vmatrix} \Rightarrow |A|\,(|A|^{2-1}) = 400 \Rightarrow (|A|)^2 = (20)^2 \Rightarrow |A| = 20$

118.(D)

Let $\quad I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\operatorname{cosec} x \cot x}{1+\operatorname{cosec}^2 x}\,dx$

Put $\operatorname{cosec} x = t \Rightarrow \operatorname{cosec} x \cot x = -dt$. When $x = \frac{\pi}{6}$, $t = 2$ and when $x = \frac{\pi}{2}$, $t = 1$

$\therefore \quad I = \int_2^1 (-1)\frac{dt}{1+t^2}$

$= \int_1^2 \frac{dt}{1+t^2} = \left[\tan^{-1} t\right]_1^2 = \tan^{-1}(2) - \tan^{-1}(1) = \tan^{-1}\left[\frac{2-1}{1+(2)(1)}\right] = \tan^{-1}\left(\frac{1}{3}\right)$

119.(C)

We have $\frac{dv}{dt} \propto -(4\pi r^2)$

We know that $v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$.

$\therefore \quad 4\pi r^2 \frac{dr}{dt} \propto -(4\pi r^2)$

$\therefore \quad 4\pi r^2 \frac{dr}{dt} = (-4k\pi r^2) \Rightarrow \frac{dr}{dt} = -k$

$\therefore \quad \int dr = -k\int dt$

$\therefore \quad r = -kt + c \qquad \ldots(1)$

We have $r = 3$, when $t = 0$

$\therefore \quad 3 = c \Rightarrow r = -kt + 3 \qquad \ldots(2)$

We have $r = 2$, when $t = 1$

$\therefore \quad 2 = -k + 3 \Rightarrow k = 1 \qquad \ldots(3)$

$\therefore \quad r = -t + 3 \Rightarrow r = 3 - t \qquad \ldots$[From (1), (2), (3)]

120.(D)

We have $y^2 = 4ax$

$\therefore\ 2y\frac{dy}{dx} = 4a \quad \Rightarrow a = \left(\frac{y}{2}\right)\frac{dy}{dx}$

$\therefore\ y^2 = 4\left(\frac{y}{2}\right)\left(\frac{dy}{dx}\right)x \quad \Rightarrow y^2 = 2xy\frac{dy}{dx}$

$\therefore\ 2x\frac{dy}{dx} - y = 0$

121.(A)

For given coplanar vectors, we write

$$\begin{vmatrix} 2 & -1 & -1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0 \Rightarrow 2(10+3\lambda) + (14) - (\lambda - 6) = 0 \Rightarrow 5\lambda + 40 = 0 \Rightarrow \lambda = -8$$

122.(C)

$\lim_{x\to 2}(x-1)^{\frac{1}{3x-6}}$

$= \lim_{x\to 2}(x-2+1)^{\frac{1}{3(x-2)}} = \lim_{x\to 2}\left\{\left[1+(x-2)\right]^{\frac{1}{(x-2)}}\right\}^{\frac{1}{3}} = e^{\frac{1}{3}}$

123.(D)

We will write logical form of given statements

$S_1 = p \rightarrow q$ $\qquad S_2 = p \rightarrow \sim q$

$S_3 = \sim p \vee q$ $\qquad S_4 = p \wedge q$

We know that $p \rightarrow q \equiv \sim q \rightarrow \sim p \equiv \sim(\sim q) \vee \sim p \equiv q \vee \sim p$

$\therefore\ S_1 = S_3$

124.(B)

We have $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\therefore\ \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$

$\therefore\ \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x} \quad \Rightarrow \sin x\cos x = \sin^2 x$

$\therefore\ \sin x(\sin x - \cos x) = 0 \quad \Rightarrow \sin x = 0 \quad$ or $\quad \tan x = 1$

$\therefore\ x = 0 \quad$ or $\quad x = \frac{\pi}{4}$

125.(C)

Let $I = \int \frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}dx$

Put $\sqrt{x} = t \quad \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\therefore \quad I = 2\int \tan^4 t \sec^2 t \, dt$

Put $\tan t = u \quad \Rightarrow \quad \sec^2 t \, dt = du$

$\therefore \quad I = 2\int u^4 \, du$

$$= \frac{2u^5}{5} = \frac{2(\tan t)^5}{5} = \frac{2}{5}\tan^5\sqrt{x} + c$$

126.(C)

Line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in plane $x + 3y - \alpha z + \beta = 0$

$\therefore \quad (3)(1) + (-5)(3) + 2(-\alpha) = 0 \quad \Rightarrow \quad \alpha = -6$

Thus equation of plane is $x + 3y + 6z + \beta = 0$ and point $(2, 1, -2)$ lies in it.

$\therefore \quad 2 + 3(1) + 6(-2) + \beta = 0 \Rightarrow \beta = 7$

$\therefore \quad \alpha\beta = (-6)(7) = -42$

127.(B)

$(p \wedge q) \wedge [(p \wedge q) \vee (\sim p \wedge q)]$

$\equiv (p \wedge q) \wedge [q \wedge (p \vee \sim p)]$

$\equiv (p \wedge q) \wedge [q \wedge T] \quad \equiv (p \wedge q) \wedge q \quad \equiv p \wedge q$

128.(C)

$\overline{PQ} = -\hat{i} + (y-5)\hat{j} + (4-x)\hat{k}$

$\overline{PR} = \hat{i} + 3\hat{j} - x\hat{k}$

Since points P, Q, R are collinear,

$\overline{PQ} = a\,\overline{PR}$

$\therefore \quad -\hat{i} + (y-5)\hat{j} + (4-x)\hat{k} = a\left(\hat{i} + 3\hat{j} - x\hat{k}\right)$

$\therefore \quad a = -1,\ 3a = y - 5,\ -ax = 4 - x$

$\therefore \quad a = -1 \quad \Rightarrow -3 = y - 5 \quad$ i.e. $y = 2$ and $x = 4 - x \Rightarrow x = 2 \Rightarrow x + y = 4$

129.(C)

$$\frac{dy}{dx} = \frac{y+1}{x^2 - x}$$

$$\therefore \int \frac{dy}{y+1} = \int \frac{dx}{x(x-1)}$$

$$\therefore \int \frac{dy}{y+1} = \int \left[\frac{1}{x-1} - \frac{1}{x}\right] dx \Rightarrow \log|y+1| = \log|x-1| - \log|x| + \log c$$

We have $x = 2$ and $y = 1$

$\therefore \quad \log|2| = \log|1| - \log|1| + \log c \quad \Rightarrow c = 2$

$\therefore \quad \log|y+1| = \log|x-1| - \log|x| + \log|2|$

$$\log|y+1| = \log\left|\frac{2(x-1)}{x}\right|$$

$$\therefore \quad y + 1 = \frac{2(x-1)}{x} \quad \Rightarrow xy + x = 2x - 2 \quad \Rightarrow xy = x - 2$$

130.(C)

We have

x	1	2	3	4	5	6
F(X = x)	0.2	0.37	0.48	0.62	0.85	1

$\therefore$ $P(x=1)=0.2,\ P(x=2)=0.17,\ P(x=3)=0.11,$
$P(x=4)=0.14,\ P(x=5)=0.23,\ P(x=6)=0.15$

Thus $P(x=4)+P(x=5)=0.14+0.23=0.37$

131.(C)

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Put $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = \frac{x+vx}{x-vx} = \frac{1+v}{1-v}$$

$$\therefore x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v} = \frac{1+v^2}{1-v}$$

$$\therefore \int \frac{(1-v)}{1+v^2}dv = \int \frac{dx}{x}$$

$$\therefore \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$

$$\therefore \tan^{-1}(v) - \frac{1}{2}\log|1+v^2| = \log|x| + c$$

$$\therefore \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right| - \frac{1}{2}\log|x^2| = c$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\left[\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x^2|\right] = c$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log|x^2+y^2| = c$$

132.(D)

Equation of required line is

$\frac{x+2}{-2} = \frac{y+2}{2} = \frac{z-3}{-1}$ and this line meets YZ plane in P.

Coordinates of any point on this line are $(-2\lambda - 2, 2\lambda - 2, -\lambda + 3)$, where λ is a scalar.

Since P is on YZ plane, we write

$-2\lambda - 2 = 0 \Rightarrow \lambda = -1$

$\therefore P \equiv (0, -4, 4)$

133.(B)

Probability of drawing red ball from first bag $= \frac{3}{8}$ and black ball $= \frac{5}{8}$

Similarly probability of drawing red ball from second bag $= \frac{6}{10}$ and black ball $= \frac{4}{10}$

$\therefore$ Required probability $= \left(\frac{3}{8}\times\frac{4}{10}\right)+\left(\frac{5}{8}\times\frac{6}{10}\right) = \frac{12+30}{80} = \frac{21}{40}$

134.(C)

$$AB = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 & 2+2+2 \\ -1-3+0 & -2+1+6 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ -4 & 5 \end{bmatrix}$$

$\therefore$ $|AB| = -25 + 24 = -1$

$$\text{adj}(AB) = \begin{bmatrix} 5 & -6 \\ 4 & -5 \end{bmatrix}$$

$$\therefore (AB)^{-1} = \frac{\begin{bmatrix} 5 & -6 \\ 4 & -5 \end{bmatrix}}{(-1)} = \begin{bmatrix} -5 & 6 \\ -4 & 5 \end{bmatrix}$$

135.(A)

We know that $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\therefore$ a sin B = b sin A ...(1) and we have

a cos B = b cos A ...(2)

From (1) and (2), we write

$\frac{a}{b} = \frac{\sin A}{\sin B} = \frac{\cos A}{\cos B} \Rightarrow \tan A = \tan B \Rightarrow A = B$

Thus Δ is an isosceles triangle.

136.(D)

A coin is tossed 4 times.

$\therefore$ $n(S) = 2^4 = 16$

Following possibilities exist.

(i) All Heads $\Rightarrow$ 1 way

(ii) 3 Heads, 1 Tail $\Rightarrow \frac{4!}{3!} = 4$ ways

(iii) 2 Heads, 2 Tails $\Rightarrow \frac{4!}{2!2!} = 6$ ways

(iv) 1 Head, 3 Tails $\Rightarrow \frac{4!}{3!} = 4$ ways

(v) 0 Head, 4 Tails $\Rightarrow$ 1 way.

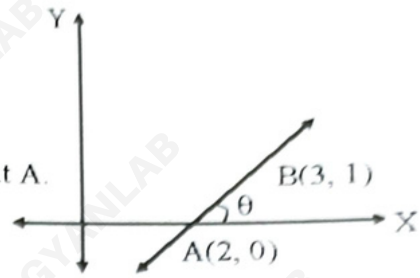
$\therefore$ Required probability.

$= P(x = 0, 1, 2) = \frac{1}{16}+\frac{4}{16}+\frac{6}{16}$

$= \frac{11}{16}$

137.(B)

Refer Figure



Slope of AB $= \dfrac{1-0}{3-2} = 1$

$\therefore \tan\theta = 1 \Rightarrow \theta = 45^\circ$

Line AB is rotated through 15° in anticlockwise direction about A.

Therefore in a new position, slope of line

$= \tan(45^\circ + 15^\circ) = \tan 60^\circ = \sqrt{3}$ and it passes through A.

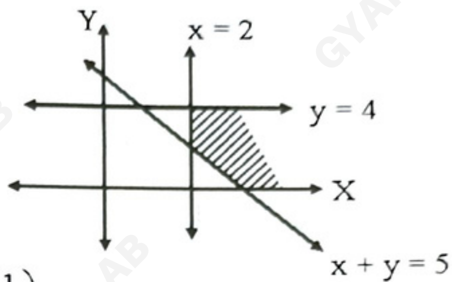
Required equation of line is $(y-0) = \sqrt{3}(x-2) \Rightarrow \sqrt{3}x - 2\sqrt{3} = y$

138.(A)

Refer figure

Required area is shaded.

It is unbounded and non-origin side.



139.(A)

We have np = 4 and npq = 2

$\therefore q = \dfrac{2}{4} = \dfrac{1}{2} \Rightarrow p = 1 - \dfrac{1}{2} = \dfrac{1}{2}$ and $n\left(\dfrac{1}{2}\right) = 4 \Rightarrow n = 8$

$\therefore P(x=2) = {}^8C_2 \left(\dfrac{1}{2}\right)^2\left(\dfrac{1}{2}\right)^6 = \dfrac{8!}{2!6!}\left(\dfrac{1}{2}\right)^8 = \dfrac{8\times 7}{2\times 2^3 \times 2^5} = \dfrac{7}{64} = \dfrac{28}{256}$

140.(D)

We have $4x - 3z + 5 = 0$ and $y = 2$

$\therefore 4x = 3z - 5 \Rightarrow 4x = 3\left(z - \dfrac{5}{3}\right)$

$\therefore \dfrac{4x}{12} = \dfrac{3\left(z-\dfrac{5}{3}\right)}{12} \Rightarrow \dfrac{x}{3} = \dfrac{\left(z - \dfrac{5}{3}\right)}{4}$

Thus line passes through point $(0, 2, \dfrac{5}{3})$, and has direction ratios 3, 0, 4.

Hence required equation of line is $\left(2\hat{j} + \dfrac{5}{3}\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{k}\right)$.

141.(A)

$x^y \cdot y^x = 16$

Taking log on both sides, $\therefore y\log x + x\log y = \log 16$

Differentiating w.r.t. x, $\therefore \dfrac{y}{x} + (\log x)\dfrac{dy}{dx} + \left(\dfrac{x}{y}\right)\dfrac{dy}{dx} + \log y = 0$

$\therefore \left[\log x + \dfrac{x}{y}\right]\dfrac{dy}{dx} = -\left[\dfrac{y}{x} + \log y\right]$

$\therefore \dfrac{dy}{dx} = \dfrac{-\left[\dfrac{y}{x} + \log y\right]}{\left[\log x + \dfrac{x}{y}\right]} \Rightarrow \left(\dfrac{dy}{dx}\right)_{(2,2)} = -\left[\dfrac{(1+\log 2)}{\log 2 + 1}\right] = -1$

142.(B)

Let $I = \int_0^2 |2x-3|\,dx$

When $x = \frac{3}{2}$, $2x - 3 = 0$

$\therefore\ I = \int_0^{3/2}(3-2x)\,dx + \int_{\frac{3}{2}}^{2}(2x-3)\,dx = [3x]_0^{3/2} - \frac{2}{2}[x^2]_0^{3/2} + \frac{2}{2}[x^2]_{3/2}^{2} - [3x]_{3/2}^{2}$

$= \left(\frac{9}{2}\right) - \left(\frac{9}{4}\right) + \left(4 - \frac{9}{4}\right) - 3\left(2 - \frac{3}{2}\right) \quad = \frac{5}{2}$

143.(D)

let $I = \int \cos^{-1}x\,dx = \int(\cos^{-1}x)\cdot(1)\,dx$

$= x\cos^{-1}x - \int \frac{-x}{\sqrt{1-x^2}}dx \quad = x\cos^{-1}x - \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}dx$

Put $\sqrt{1-x^2} = t \quad \Rightarrow \frac{(1)(-2x)}{2\sqrt{1-x^2}}dx \quad = dt$

$= x\cos^{-1}x - \int dt \ = x\cos^{-1}x - t \ = x\cos^{-1}x - \sqrt{1-x^2} + c$

144.(C)

Let $I = \int \frac{1}{\cos x + \sqrt{3}\sin x}dx$

Dividing numerator and denominator by 2, we get

$= \frac{1}{2}\int \frac{dx}{\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)} \quad = \frac{1}{2}\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)} \quad = \frac{1}{2}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)\right| + C$

145.(D)

$f(x) = [8x] - 3$

$\therefore\ f(\pi) = [8(3.14)] - 3 = [25.12] - 3 = 25 - 3 = 22$

146.(C)

Lines represented by $px^2 - qy^2 = 0$ are distinct.

Here $h = 0$, $a = p$ and $b = -q$

Now $h^2 - ab > 0 \Rightarrow 0 + pq > 0 \quad \Rightarrow pq > 0$

147.(D)

Volume of cone $= \frac{1}{3}\pi r^2 h$

We have slant height $\ell = 3$ cm and we know that $\ell^2 = r^2 + h^2$

$\therefore\ 9 = r^2 + h^2 \quad \Rightarrow \quad r^2 = 9 - h^2$

$\therefore\ v = \frac{1}{3}\pi(9 - h^2)h \quad = (3\pi)h - \left(\frac{\pi}{3}\right)h^3$

$$\frac{dv}{dh} = 3\pi - \left(\frac{\pi}{3}\right)(3h^2) = 3\pi - \pi h^2$$

When $\frac{dv}{dh} = 0$, we get $3\pi = \pi h^2 \Rightarrow h = \sqrt{3}$

$$\frac{d^2v}{dh^2} = 0 - 2\pi h \Rightarrow \left(\frac{d^2v}{dh^2}\right)_{h=\sqrt{3}} = -2\sqrt{3}\,\pi < 0$$

$\therefore$ Volume of cone is maximum when $h = \sqrt{3}$.

148.(D)

$$(3+5\omega+3\omega^2)^2 + (3+3\omega+5\omega^2)^2$$

$$= (3+3\omega+3\omega^2+2\omega)^2 + (3+3\omega+3\omega^2+2\omega^2)^2$$

$$= \left[3(1+\omega+\omega^2)+2\omega\right]^2 + \left[3(1+\omega+\omega^2)+2\omega^2\right]^2$$

$$= (3(0)+2\omega)^2 + \left[3(0)+2\omega^2\right]^2 = 4\omega^2 + 4\omega^4 = 4\omega^2(1+\omega^2) = 4\omega^2(-\omega)$$

$$= -4\omega^3 = -4$$

149.(C)

We have $p, q \equiv T$ and $r, s \equiv F$

$a : \sim(p \wedge \sim r) \vee (\sim q \vee s) \equiv \sim(T \wedge \sim F) \vee (\sim T \vee F) \equiv \sim(T \wedge T) \vee (F \vee F)$

$\equiv \sim T \vee F \equiv F \vee F \equiv F$

$b : (p \vee s) \leftrightarrow (q \wedge r) \equiv (T \vee F) \leftrightarrow (T \wedge F) \equiv T \leftrightarrow F \equiv F$

150.(A)

We have $\int_0^a \sqrt{\frac{a-x}{x}}\,dx = \frac{k}{2}$

Put $x = a\sin^2\theta \Rightarrow dx = a(2\sin\theta)\cos\theta\,d\theta = 2a\sin\theta\cos\theta\,d\theta$

When $x = 0, \theta = 0$ and when $x = a, \theta = \frac{\pi}{2}$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{\frac{a - a\sin^2\theta}{a\sin^2\theta}}\,(2a\sin\theta\cos\theta)\,d\theta = \frac{k}{2}$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{\cos^2\theta}{\sin^2\theta}}\,2(a\sin\theta\cos\theta)\,d\theta = \frac{k}{2} \Rightarrow \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta}(2a\sin\theta\cos\theta)\,d\theta = \frac{k}{2}$$

$$\int_0^{\frac{\pi}{2}} 2a\cos\theta^2\,d\theta = \frac{k}{2} \Rightarrow 2a\int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\,d\theta = \frac{k}{2}$$

$$a\int_0^{\frac{\pi}{2}} (1+\cos 2\theta)\,d\theta = \frac{k}{2} \Rightarrow a\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} = \frac{k}{2}$$

$$a\left[\left(\frac{\pi}{2}+0\right) - (0+0)\right] = \frac{k}{2} \Rightarrow a\frac{\pi}{2} = \frac{k}{2} \Rightarrow k = \pi a$$